IB MYP YEAR 5

YEAR 10 Mathematics

Assessment #1

POLYNOMIALS

Name: Elizabeth Kot (10 Trust.)

Teacher: **Ms. Li, Mr. So & Mr. Wong**

Date of task: **Friday, October 5, 2013** Time allowed: **60 minutes**

Student's **Performance** in Different **Criterion**			
A	4	**C**	4

dli

INSTRUCTIONS:

- Read the instructions for all questions carefully.
- Show all work, steps and proper units.
- Ask the teacher for scrap paper, but any work on the scrap paper will **NOT** be marked.
- Write in **PENCIL**.
- **NOT** allowed to use any **electronic devices**, such as translators.
- Allowed to use **calculators**
- Allowed to use **non-electronic dictionary**.

ASSESSMENT:

- Read the criteria descriptors carefully before you start your work. This will give you a clear understanding of what is required and what a quality piece of work for this task must include. This way you give yourself the best chance of achieving the highest level in this task.
- This task assesses Criteria **A & C**.
  - For Criteria **A**, the questions are all assigned with levels;
  - Criterion **C** will be assessed as an **overall impression** on the presentation of work in this assessment.

Criterion A: KNOWLEDGE AND UNDERSTANDING

Achievement level	Task Specific Rubric	IBO Published Descriptor	Student's self-evaluation
0	The student does not reach a standard described by any of the descriptors given below.	The student does not reach a standard described by any of the descriptors given below.	
1–2 **Simple**	The student can solve **some simple** problems.	The student **attempts** to make deductions when solving **simple** problems in **familiar** contexts.	**(0-8)**
3–4 **Complex**	The student can solve **most simple** and **some more complex** problems.	The student **sometimes** makes **appropriate** deductions when solving **simple and more-complex** problems in **familiar** contexts	**Teacher's Final Grade**
5–6 **Challenging**	The student can solve **challenging** problem correctly and **most** familiar problems along with **all** different types of problems.	The student **generally** makes **appropriate** deductions when solving **challenging** problems in a **variety** of **familiar** contexts.	
7–8 **Unfamiliar**	The student can solve **most** challenging and **most** familiar problems along with **all** different types of problems.	The student **consistently** makes **appropriate** deductions when solving **challenging** problems in a **variety** of contexts including **unfamiliar** situations.	**(0-8)**

Criterion C: COMMUNICATION IN MATHEMATICS

Achievement level	Task Specific Rubric	IBO Published Descriptor	Student's self-evaluation
0	The student does not reach a standard described by any of the descriptors given below.	The student does not reach a standard described by any of the descriptors given below.	
1–2	The student should be able to explain **some problems** step by step. The lines of reasoning are **difficult to follow**.	• The student shows **basic** use of mathematical language **and/or** forms of mathematical representation. • The lines of reasoning are **difficult to follow**.	**(0-6)**
3–4	The student should be able to explain **most problems** step by step. The lines of reasoning are **clear** though **not always logical** or **complete**.	• The student shows **sufficient** use of mathematical language **and** forms of mathematical representation. • The lines of reasoning are **clear** though not always **logical** or **complete**. • The student moves between different forms of representation **with some success**.	**Teacher's Final Grade**
5–6	The student should be able to explain **most problems** step by step. The lines of reasoning are **concise**, **logical** and **complete**. The student use **correct unit** in the questions.	• The student shows **good** use of mathematical language **and** forms of mathematical representation. • The lines of reasoning are **concise**, **logical** and **complete**. • The student moves **effectively** between different forms of representation.	**(0-6)**

A. Simple problems

The suggested time allocated for **Question 1 to 4 is 12 minutes**.

1. Solve each of the following equations.

(a) $3(x-2)(x+1)=0$

$(3x-6)(3x+3)$
$=9x^2+9x-18x-18$
$=9x^2-9x-18.$

$=(3x+6)(3x-3)$

$3x+6=0$ OR $3x-3=0$
$x=\frac{-6}{3}$ $x=\frac{3}{3}$
$x=-2$ $x=1$

(b) $2(x-1)^2-6=0$

$(2x-1)^2-6=0$
$4x^2-4x+1-6=0$
$4x^2-4x-5=0$

$x=\frac{+4\pm\sqrt{(-4)^2-4(4)(-5)}}{2(4)}$
$=\frac{4\pm\sqrt{96}}{8}$
$=\frac{1\pm\sqrt{96}}{2}.$

(c) $3x^2-2x-1=0$

$x=\frac{+2\pm\sqrt{(-2)^2-4(3)(-1)}}{2(3)}$

$x=\frac{2\pm\sqrt{4+12}}{6}$

$x=\frac{2\pm\sqrt{16}}{6}$
$=\frac{2\pm4}{6}$

$x=\frac{2+4}{6}$ OR $x=\frac{2-4}{6}$
$=\frac{6}{6}$ $x=\frac{-2}{6}$
$=1$ $=-\frac{1}{3}$

2. Find the remainder when $f(x)=x^3+x^2+3x+4$ is divided by $(x-1)$.

$f(1)=1^3+1^2+3(1)+4$
$=1+1+3+4$
$=9.$

3. If $f(x)=x^4-2x^2+k$ is divisible by $(x+2)$, find k.

$f(-2)=(-2)^4-2(-2)^2+k$
$=16-8+k$
$=8+k.$ $k=-8$

4. If the function of a parabola is $y=2(x+1)^2-5$, find the vertex of the parabola.

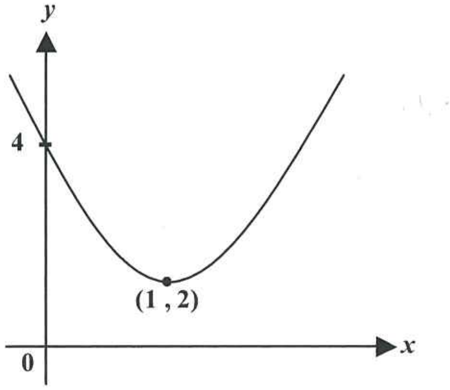
$(-1,-5)$

B. More complex problems

The suggested time allocated for **Question 5 to 7 is 18 minutes**.

5. Find the functions of each of the following parabolas.

(a)



where **(1,2)** is the **vertex** of the above parabola.

$y = a(x-h)^2+k$

$4 = a(x-1)^2+2$ — ①

Sub $x=0$ into ①

$4 = a(0-1)^2+2$

$4 = a(0^2+2(0)(-1)+(-1)^2)+2$

$4 = a+2$

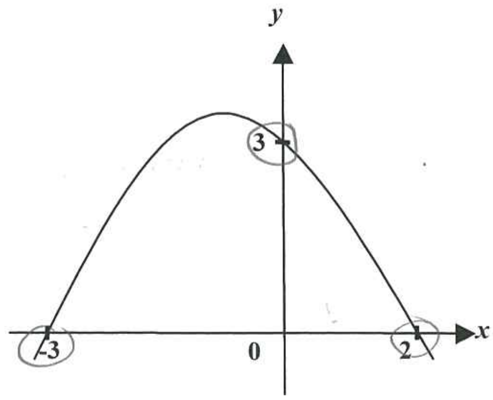
$-a = 2-4$

$-a = -2$

$a = 2$.

The function of the graph is $y = 2(x-1)^2+2$.

(b)



$y = a(x-h)^2+k$

$3 = a(x+3)^2+k$ — ①

$3 = a(x-2)^2+k$ — ②

Sub $x=0$ into ①

$3 = a(0+3)^2+k$

$3 = a(0^2+2(0)(3)+3^2)+k$

$3 = 9a+k$ — ③

Sub $x=0$ into ②

$3 = a(0-2)^2+k$

$3 = 4a+k$ — ④

③ − ④

$3 = 9a+k$

$-)\ 3 = 4a+k$

$0 = 5a$

$a = -5$.

$y = -5(x+3)^2+k$ $(k=\frac{45}{3})$

or

$y = -5(x-2)^2+k$. $(k=\frac{20}{3})$

6. The polynomial $f(x) = x^3 + ax^2 + bx - 3$ is **divisible** by $(x - 3)$. When $f(x)$ is **divided** by $(x + 2)$, the remainder is 15. Find the value of **a** and **b**.

$f(x) = x^3 + ax^2 + bx - 3$

$f(3) = 3^3 + a(3)^2 + b(3) - 3$

$= 27 + 9a + 3b - 3$

$= 24 + 9a + 3b$

$9a + 3b = -24$ — ①

$f(-2) = 15$

$(-2)^3 + a(-2)^2 + b(-2) - 3 = 15$

$-8 + 4a - 2b - 3 = 15$

$4a - 2b = 26$ — ②

① ÷ 3 — $3a + b = -8$ — ③

② ÷ 2 — $2a - b = 13$ — ④

③ + ④

$3a + b = -8$

+) $2a - b = 13$

$5a + 0 = 5$

$a = 1$

Put $a = 1$ into ②

$4(1) - 2b = 26$

$-2b = 26 - 4$

$b = \frac{22}{-2}$

$b = 11$

sub. $b = 11$ into ②

$4a - 2(11) = 26$

$4a = 26 + 22$

$4a = 48$

$a = 12$

7. If the graph of $y = mx^2 + 12x + 8$ intersects the x-axis and **m is a <u>positive integer</u>**,

(a) find the FOUR possible values of m.

(b) when m is a **maximum**, find the **roots** of the equation $mx^2 + 12x + 8 = 0$.

C. Challenging problem

The suggested time allocated for **Question 8 is 15 minutes**.

8. Given that the equation $(2k - 1)x^2 + (k - 3)x - 2 = 0$ where $k \neq \frac{1}{2}$,

(a) find the value of k if the **sum of roots equals the product of roots** of the equation.

$(2k-1)x^2 + (k-3)x - 2 = 0$

Sum of roots $= \frac{-(k-3)}{(2k-1)}$ $= -\frac{b}{a}$

$= \frac{-k+3}{2k-1}$

Product of roots $= \frac{-2}{(2k-1)}$ $= \frac{c}{a}$.

$\because$ SOR = POR

$\therefore \frac{-k+3}{2k-1} = \frac{-2}{(2k-1)}$

$\frac{-k+3}{(2k-1)} - \frac{-2}{(2k-1)} = 0$

$\frac{-k+3+2}{2k-1} = 0$

$\frac{-k+5}{2k-1} = 0$

$\frac{5-k}{2k-1} = 0$.

$5-k = 2k-1$

$-k-2k = -1-5$

$-3k = -6$

$k = 3$

(b) By using the value of **k obtained in (a)**, find the **axis of symmetry**, **x-intercept(s)** and **y-intercept**.

$(2(3)-1)x^2 + ((3)-3)x - 2 = 0$

$(6-1)x^2 + (3-3)x - 2 = 0$

$5x^2 + 0x - 2 = 0$

$5x - 2 = 0$

$5x = 2$

$x = \frac{2}{5}$

$(2x-5)$

$h = \frac{-0}{5}$

$h = -\frac{1}{5}$.

$k = \frac{4ac-b^2}{4a}$

$= \frac{4(5)(-2)-0^2}{4(5)}$

$= \frac{-40}{20}$

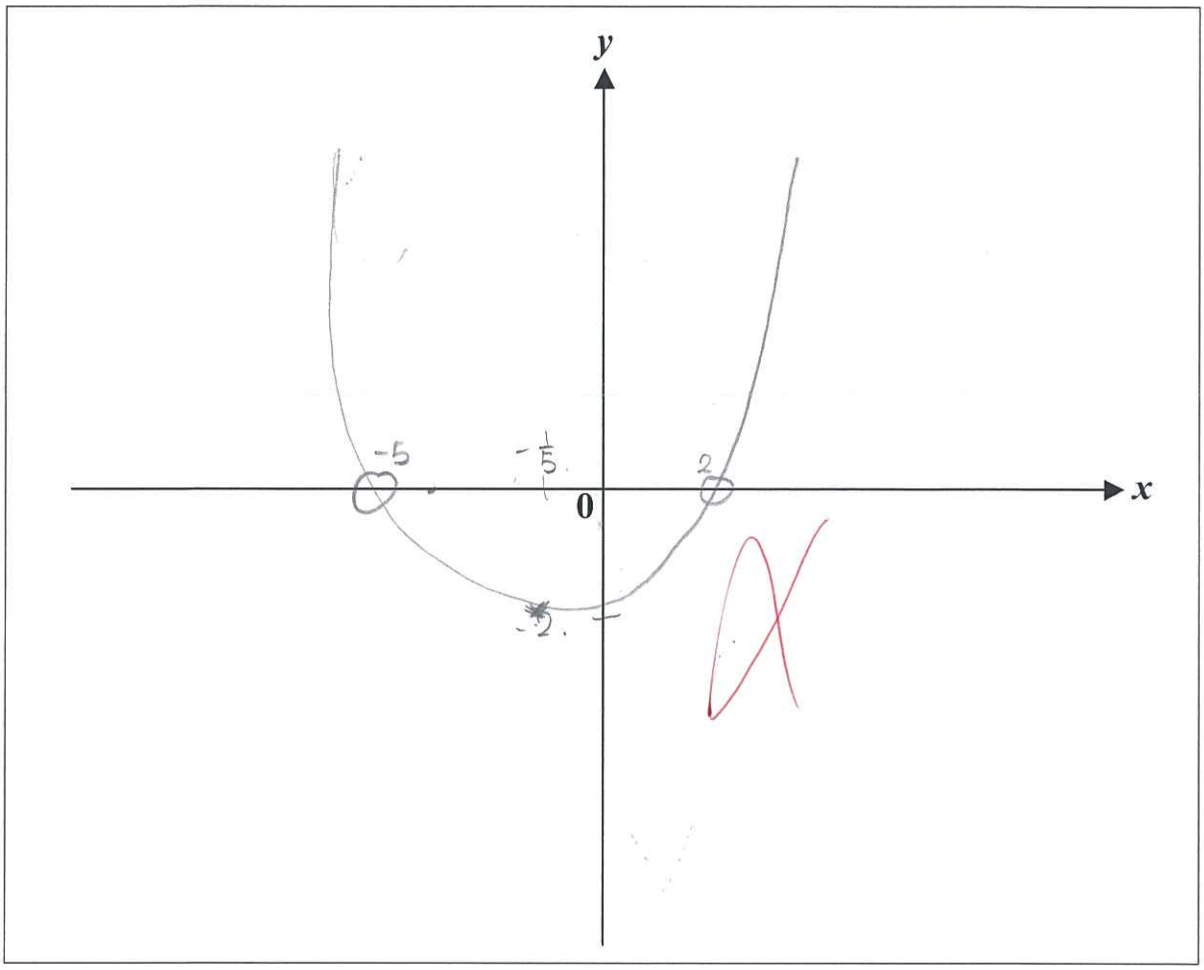
$= -2$

(h, k)

$(-\frac{1}{5}, -2)$

20/20

(c) By using the results **obtained in (b)**, sketch the function $y = (2k - 1)x^2 + (k - 3)x - 2$. The **axis of symmetry, x-intercept(s) and y-intercept** should be clearly shown on your graph.



D. Unfamiliar problems

The suggested time allocated for **Question 9 is 15 minutes**.

9. A courier company is responsible for delivering documents to Mainland China. Suppose the volume (in cm^3) of the rectangular carton used for delivery is given by $\mathbf{C(x) = x^3 - 180x^2 + 10700x - 210000}$ (where $\mathbf{x > 70}$).

MTR is the main means of transport used by that company. In order to reduce the cost, the cartons used must conform to the restrictions on the size of luggage carried by passengers on the MTR: **the sum of the length, width and height** of the luggage **should not exceed 170 cm, and the length of any side of the luggage should not exceed 130 cm.**

(a) Show that $\mathbf{x - 70}$ is a factor of $C(x)$.

$C(70) = (70)^3 - \cancel{180}(70)^2 + 10700(70) - 210000$

$=$

?

(b) Factorize $C(x)$.

(c) (i) Find the value of $C(\mathbf{110})$.

(ii) Using the results of **(b) and (c)(i)**, suggest the dimensions of a carton, with volume $\mathbf{120000cm^3}$, which conform to the restrictions on the size of luggage carried by passengers on the MTR.

End of Assessment